International Journal of Theoretical Physics, Vol. 26, No. 10, 1987

Dynamics of Electrically Charged Extended Bodies: Classical and Quantum Systems

Terje Aaberge[1]

Received June 2, 1987

We present generalizations of classical mechanics and quantum mechanics that make it possible to describe N charged extended bodies. In particular, we are able to write down a set of coupled equations for the system of N bodies plus field. The theory is based on a theory for the description of N charged chemical fluid components.

1. INTRODUCTION

How can we justify theoretically the use of the Coulomb potential in atomic physics? This is in essence the question motivating our research, the result of which is presented in this paper. The answer could be that one should write down a self-consistent set of Maxwell plus Schrödinger equations and then prove that under reasonable boundary conditions the Coulomb potential appears (at least as an approximation) in a solution of this system.

There is an immediate obstacle to the formulation of such a system of equations. In fact, Maxwell's electromagnetic field equations are "Eulerian," i.e., the field quantities are functions on space $X = \mathbb{R}^3$, while the particle dynamical equations are "Lagrangian," i.e., the configurations of an N-body system is described by N-tuples $(x_1^i, \ldots, x_N^i) \in X^N$. To surmount this difficulty we have assumed that a charged N-body system can be given a Eulerian description as a system of N charged chemical fluid components (Aaberge, 1986a,b, 1987) associated with a set of boundary conditions that are such that the chemical components are spatially separated.

This approach also solves another problem, that of finding candidates for what should be the charge and current densities for an N-body quantum

[1]Département de Physique Théorique, Université de Genève, CH-1211 Geneva 4, Switzerland. Permanent address: N-5801 Sogndal, Norway.

0020-7748/87/1000-0991$05.00/0

system. They are simply introduced as new independent quantities. This procedure is justified by the success of the application of the Coulomb potential in the construction of models of atoms, which indicates that even an electron or a proton sees the other particle in the hydrogen atom as a point particle, whatever the state of the atom. This rules out a possibility that otherwise might have been suggested, that the charge densities are related to the norm squared of the wave function.

The state space of a continuum system is a Banach manifold $\mathscr{B}(\mathscr{E})$ of sections $\gamma: X = \mathbb{R}^3 \to \mathscr{E}$ of a fibered manifold $\pi: \mathscr{E} \to X$. The (extensive) observables of the system are functions $F: \mathscr{B}(\mathscr{E}) \to \mathbb{R}$, represented as integrals $F(\gamma) = \int_\gamma f d^3x$, where $f: J_q(\mathscr{E}) \to \mathbb{R}$ is a function on the q-jet extension $J_q(\mathscr{E})$ of $\mathscr{E}$, i.e., of γ and its derivatives up to order q. Finally, the equations of motion are represented by ordinary differential equations, i.e., by vector fields on $\mathscr{B}(\mathscr{E})$. These vector fields are specified by a symbol χ whose components are functions on $J_q(\mathscr{E})$, and a set of boundary conditions.

The local extensive observables of the fluid component n are the momentum density π_{ni}, mass density ρ_n, and charge density q_n. The dynamics is assumed to be Hamiltonian, and the symbol for the dynamical vector field χ^{H} is constructed from the energy function U. As part of the boundary conditions, it is assumed that for any moment of time, the support of π_{ni} is contained in the support of ρ_n, i.e., the portion of the space occupied by the component n. Because of the separation of space that follows, the interaction between the spatially separated components must be simulated by the boundary conditions. To do this, we introduce a set of generalized irrotational conditions that permit us to establish a potential w for the local momentum. The equation of motion $w = \chi^{\mathrm{H}}(w)$ of the potential w, which is determined from the original equations of motion, is *a priori* valid only on the portion of space occupied by matter. It can, however, in "natural" ways be extended also to the void. We give two extensions. One corresponds to the classical N-body system. Then w is identified with the Hamilton–Jacobi action function, and the extension of the equation of motion is the Hamilton–Jacobi equation. The other extension considered describes the system of N quantum bodies. Then w is identified with the phase of the wave function and the extension of the equation is the equation for the phase as obtained from the Schrödinger equation.

2. DEFINITION OF THE SYSTEM

The state space $\mathscr{B}(\mathscr{E})$ of the system is *a priori* associated with the fibered manifold (Aaberge, 1986a,b; Pommaret, 1978; Palais, 1968),

$$\mathscr{E} \to X$$

where

$$\mathscr{E} = \{(x^i;\ \pi_{ni},\ \rho_n,\ q_n,\ D^i,\ B^i) \in \mathbb{R}^3 \times \mathbb{R}^{3N} \times \mathbb{R}_+^N \times \mathbb{R}^6\}$$

$$X = \{(x^i) \in \mathbb{R}^3\}$$

but *a posteriori* subjected to the constraints

$$\alpha = 0, \qquad \beta = 0, \qquad \gamma_n = 0, \qquad \Delta_{nij} = 0$$

for

$$\hat{\alpha}: \quad J_1(\mathscr{E}) \to \mathbb{R}; \quad (\cdot) \mapsto D^i{}_{,i} - \sum_n q_n$$

$$\hat{\beta}: \quad J_1(\mathscr{E}) \to \mathbb{R}; \quad (\cdot) \mapsto B^i{}_{,i}$$

$$\hat{\gamma}_n: \quad \mathscr{E} \to \mathbb{R}; \quad (\cdot) \mapsto \frac{1}{m_n}\rho_n - \frac{1}{e_n} q_n$$

$$\hat{\Delta}_{nij}: \quad J_1(\mathscr{E}) \to \mathbb{R}; \quad (\cdot) \mapsto \nabla_j \frac{\pi_{ni}}{\rho_n} - \nabla_i \frac{\pi_{nj}}{\rho_n} - \frac{e_n}{m_n} \varepsilon_{ijk} B^k$$

where $m_n = \int_X \rho_n \, d^3x$ and $e_n = \int_X q_n \, d^3x$ are the total mass and the total charge of the nth body, respectively. The local extensive observables of momentum density π_{ni}, mass density ρ_n, and charge density q_n of the nth body and of the electric displacement D^i and magnetic induction B^i are represented by the functions $\mathscr{E} \to \mathbb{R}$,

$$\hat{\pi}_{ni}(\cdot) = \pi_{ni}, \qquad \hat{\rho}_n(\cdot) = \rho_n, \qquad \hat{q}_n(\cdot) = q_n, \qquad \hat{D}^i(\cdot) = D^i, \qquad \hat{B}^i(\cdot) = B^i$$

According to the laws of thermodynamics, any (thermodynamic) system is associated with an energy function, i.e., in this setting, with an energy density (Aaberge, 1986b)

$$\hat{u}: J_q(\mathscr{E}) \to \mathbb{R}$$

To give $\hat{u}$ is equivalent to giving a representation of the intensive observables of velocity v_n^i, chemical potential μ_n, and electric potential V_n of the nth component, electric field E_i, and magnetic field H_i. They are represented by the functions $J_q(\mathscr{E}) \to \mathbb{R}$,

$$\hat{v}_n^i(\cdot) = \nabla_{\pi_{ni}} \hat{u}(\cdot), \qquad \hat{\mu}_n(\cdot) = \nabla_{\rho_n} \hat{u}(\cdot), \qquad \hat{V}_n(\cdot) = -\nabla_{q_n} \hat{u}(\cdot)$$

$$\hat{E}_i(\cdot) = \nabla_{D^i} \hat{u}(\cdot), \qquad \hat{H}_i(\cdot) = \nabla_{B^i} \hat{u}(\cdot)$$

where ∇ denote the "functional" derivative, i.e.,

$$\nabla_y = \partial_y - \nabla_i \partial_{y_{,i}} + \nabla_i \nabla_j \partial_{y_{,ij}} - \cdots$$

and $y_{,i}$ corresponds to the variable representing the partial derivative of $y(x^i)$,

$$y_{,i}(x^i) = \partial_{x^i} y(x^i)$$

3. THE DYNAMICS

Let $\tilde{\mathscr{E}}$ be the fibered manifold

$$\tilde{\mathscr{E}} = \{(x^i; u_{ni}, \varphi^i_n, D^i, A^i) \in \cdots\} \to X$$

and let ω denote a symplectic form on the fibers, associated with the symbol

$$\omega = \sum_n du_{ni} \wedge d\varphi^i_n + dA_i \wedge dD^i$$

Moreover, denote by Ψ the submersion

$$\begin{aligned}
&\{J_1(\tilde{\mathscr{E}})\,|\,j_{\varphi_n} = \det(\varphi^i_{n,j}) \neq 0\} \\
&\quad \to E = \{\mathscr{E}\,|\,\alpha = 0, \beta = 0, \gamma_n = 0, \Delta_{nij} = 0, n = 1, \ldots\} \\
&\qquad (x^i; u_{ni}, \varphi^i_n, D^i, A_i, u_{ni,j}, \varphi^i_{n,j}, D^i_{,j}, A_{i,j}) \\
&\qquad \mapsto (x^i; \varphi^j_{n,i} u_{nj} - e_n j_{\varphi_n} \varkappa_n \circ \varphi_n A_i, m_n, j_{\varphi_n} \varkappa_n \circ \varphi_n \\
&\qquad\quad D^i_{,i} - \sum_{\substack{m \\ m \neq n}} e_m j_{\varphi_m} \varkappa_m \circ \varphi_m, D^i, \varepsilon^{ijk} A_{k,j})
\end{aligned}$$

The Hamiltonian of the system is, by assumption, the function

$$\hat{h}: \quad \{J_{q+1}(\tilde{\mathscr{E}})\,|\,j_{\varphi_n} \neq 0\} \to \mathbb{R}$$

obtained by the pullback of the restriction of $J_q(\Psi)$ to $\{J_q(\mathscr{E})\,|\,j_{\varphi_n} \neq 0\}$ of the restriction of $\hat{u}$ to E.

Proposition 1. The pushforward of the Hamiltonian vector field

$$\chi^{\mathrm{H}} = (\nabla_{\varphi^i_n} \hat{h}, -\nabla_{u_{ni}} \hat{h}, \nabla_{A_i} \hat{h}, -\nabla_{D^i} \hat{h})$$

under Ψ has the components

$$\begin{aligned}
\chi^{\mathrm{H}}(\pi_{ni}) &= -\nabla_j(\pi_{ni} v^j_n) - \pi_{nj} \nabla_i v^j_n - \rho_n \nabla_i \mu_n + q_n \nabla_i V_n + q_n(E_i + \varepsilon_{ijk} v^j_n B^k) \\
\chi^{\mathrm{H}}(\rho_n) &= -\nabla_i(\rho_n v^i_n) \\
\chi^{\mathrm{H}}(q_n) &= -\nabla_i(q_n v^i_n) \\
\chi^{\mathrm{H}}(D^i) &= \varepsilon^{ijk} \nabla_j H_k - \sum_n q_n v^i_n \\
\chi^{\mathrm{H}}(B^i) &= -\varepsilon^{ijk} \nabla_j E_k
\end{aligned}$$

Proof. The result is obtained by a direct computation, noticing that

$$\begin{aligned}\nabla_{\varphi_n^i}\hat{h} &= -\nabla_j(u_{ni}v_n^j) - \nabla_j(e_n j_{\varphi_n}\varkappa_n\circ\varphi_n(\varphi_{n,i}^j)^{-1}A_k v_n^k)\\ &\quad + e_n j_{\varphi_n}(\partial_{y_n^i}\varkappa_n)\circ\varphi_n A_k v_n^k - \nabla_j[m_n j_{\varphi_n}\varkappa_n\circ\varphi_n(\varphi_{n,i}^j)^{-1}\mu_n]\\ &\quad + m_n j_{\varphi_n}(\partial_{y_n^i}\varkappa_n)\circ\varphi_n\mu_n - \sum_{\substack{m\\ m\neq n}}\nabla_j[e_m j_{\varphi_m}\varkappa_m\circ\varphi_m(\varphi_{m,i}^j)^{-1}V_m]\\ &\quad + \sum_{\substack{m\\ m\neq n}} e_m j_{\varphi_n}(\partial_{y_m^i}\varkappa_m)\circ\varphi_m V_m\\ \nabla_{u_{ni}}\hat{h} &= \varphi_{n,j}^i v_n^j\\ \nabla_{D^i}\hat{h} &= \sum_n \nabla_i V_n + E_i\\ \nabla_{A_i}\hat{h} &= \nabla_j\varepsilon^{ijk}H_k - \sum_n q_n v_n^i\end{aligned}$$

Proposition 2. The total energy, total momentum, total mass, total charge, and total magnetic flux through any closed surface are conserved.

Proof. Computation gives

$$\begin{aligned}\chi^{\mathrm{H}}(u) &= -\nabla_i[\sum_n(\pi_{nj}v_n^j + \rho_n\mu_n - q_n V_n)v_n^i + \varepsilon^{ijk}E_j H_k]\\ \chi^{\mathrm{H}}(&\sum_n \pi_{ni} + \varepsilon_{ijk}D^j B^k)\\ &= -\nabla_j\{\sum_n \pi_{ni}v_n^j + P^j{}_i - D^j E_i\\ &\quad - B^j H_i + \delta^j{}_i[-u + \sum_n(\pi_{nk}v_n^k + \rho_n\mu_n - q_n V_n) + D^k E_k + B^k H_k]\}\end{aligned}$$

where $P^i{}_j \neq 0$ only if u is effectively a function of $\pi_{ni,j}$, $\rho_{n,i}, \ldots$. The expression for $P^i{}_j$ is found in Aaberge (1986a, p. 398).

Proposition 3. The constraints are invariant under the dynamical evolution, i.e.,

$$\chi^{\mathrm{H}}(\hat{\alpha}) = 0, \qquad \chi^{\mathrm{H}}(\hat{\beta}) = 0, \qquad \chi^{\mathrm{H}}(\hat{\gamma}_n) = 0, \qquad \chi^{\mathrm{H}}(\hat{\Delta}_{nij}) = 0,$$
$$n = 1, 2, \ldots, N$$

on the subspace satisfying the constraints $\hat{\alpha} = 0$, $\hat{\beta} = 0$, $\hat{\gamma}_n = 0$, and $\hat{\Delta}_{nij} = 0$ for all n.

Proof. Direct computation.

Supplemented with the proper boundary conditions, the symbol χ^{H} defines a vector field that is supposed to describe a system of N charged bodies. It is evident that it might be difficult to construct an actual solution to such a system. However, we will justify the claim that it might be so by specifying some of the boundary conditions that one must assume.

4. THE LOCAL MOMENTUM POTENTIAL

The equations of motion associated with a vector field defined by a symbol χ^{H} and a set of boundary conditions contain in principle all the information necessary to determine the evolution of the system. In the case of a system of N spatially separated chemical components or bodies, the space X is divided into a domain $\mathscr{D}_{\mathrm{M}}$ that is filled with matter, $\mathscr{D}_{\mathrm{M}} = \bigcup_n \mathrm{supp}(\rho_n)$, and a domain $\mathscr{D}_{\mathrm{V}}$ that is "empty," $\mathscr{D}_{\mathrm{V}} = X/\mathscr{D}_{\mathrm{M}}$. *A priori* the symbol χ^{H} might seem to be nonzero on $\mathscr{D}_{\mathrm{M}}$ only, since we must chose $\pi_{ni}(x^i) = 0$, $\chi^{\mathrm{H}}(\pi_{ni})(x^i) = 0$, etc., for $x^i \in \mathscr{D}_{\mathrm{V}}$. However, χ^{H} might also be defined by its action on nonvanishing quantities on $\mathscr{D}_{\mathrm{V}}$. Thus, for example,

$$\chi^{\mathrm{H}}\left(\frac{\pi_{ni}}{\rho_n}\right) = -v_n^j \nabla_j \frac{\pi_{ni}}{\rho_n} - \frac{\pi_{nj}}{\rho_n} \nabla_i v_n^j - \nabla_i \mu_n - \frac{q_n}{\rho_n} \nabla_i V_n + \frac{q_n}{\rho_n} (E_i + \varepsilon_{ijk} v_n^j B^k)$$

On E this can be rewritten

$$\chi^{\mathrm{H}}\left(\frac{\pi_{ni}}{\rho_n}\right) = -\nabla_i \left(v_n^j \frac{\pi_{nj}}{\rho_n} + \mu_n - \frac{e_n}{m_n} V_n \right) + \frac{e_n}{m_n} E_i$$

Assume that there exists a function $\tilde{w}: \mathbb{R}^{3N} \to \mathbb{R}$ (or S^1) such that

$$\frac{u_{ni}}{\rho_n / m_n} = (\partial_{y_n^i} \tilde{w})(\varphi_1^i(x^i), \ldots, \varphi_N^i(x^i))$$

Then

$$\sum_n \frac{\varphi_{n,i}^j u_{nj}}{\rho_n / m_n} = \nabla_i \tilde{w}$$

Accordingly,

$$\nabla_i \chi^{\mathrm{H}}(\tilde{w}) = -\nabla_i \sum_n \left(v_n^j \frac{\pi_{nj}}{\rho_n / m_n} + m_n \mu_n + e_n \sum_{\substack{m \\ m \neq n}} V_m \right)$$

i.e., modulo a constant,

$$\chi^{\mathrm{H}}(\tilde{w}) = -\sum_n \left(v_n^i \frac{\pi_{ni}}{\rho_n / m_n} + m_n \mu_n + e_n \sum_{\substack{m \\ m \neq n}} V_m \right)$$

or if we specialize to the case where

$$u(\cdot) = \sum_n \frac{\delta^{ij} \pi_{ni} \pi_{nj}}{2\rho_n} + u_0(\rho_n, q_n, D^i, B^i)$$

then

$$\chi^{\mathrm{H}}(\tilde{w}) = -\sum_n \left\{ \frac{\delta^{ij}}{2m_n} [\varphi^k_{n,i}(\partial_{y_n^k}\tilde{w} - e_n A_i][\varphi^l_{n,j}(\partial_{y_n^l}\tilde{w}) - A_j] \right.$$

$$\left. + e_n \tfrac{1}{2} \sum_{\substack{m \\ m \neq n}} V_m + m_n \mu_{0n} \right\}.$$

5. THE CLASSICAL AND QUANTUM EXTENSIONS

The above expression is a formal consequence of the application of the symbol χ^{H} on the function $\tilde{w}$. It is *a priori* valid on $\mathscr{D}_{\mathrm{M}}$; however, we may assume an extension of the validity of the symbol as part of the boundary conditions. Thus we may write

$$\chi^{\mathrm{H}}(\tilde{w}) = -\sum_n \left\{ \frac{\delta^{ij}}{2m_n} [\varphi^k_{n,i}(\partial_{y_n^k}\tilde{w}) - e_n A_{ni}][\varphi^l_{n,j}(\partial_{y_n^l}\tilde{w}) - e_n A_{nj}] \right.$$

$$\left. + e_n \frac{1}{2} \sum_{\substack{m \\ m \neq n}} V_{m,n} + m_n f_n \right\}$$

The potentials A_{ni} are assumed to be solutions of the Maxwell equations on $\mathscr{D}_{\mathrm{V}}$, i.e.,

$$\partial_t A_i = -E_i - \nabla_i \sum_n V_n, \qquad \partial_t D^i = \varepsilon^{ijk} \nabla_j H_k$$

where

$$E_i = \frac{\delta_{ij}}{\varepsilon_0} D^i, \qquad H_i = \mu_0 \delta_{ij} B^j$$

and $A_{ni} = A_i$ on $\partial\,\mathrm{supp}(q_n)$. Some extra information is needed to establish the potentials $(V_{m,n})$; in fact, we must evoke the definition of electric field strength, extending the definition of $V_m = -\nabla_{q_m} u$. Here V_m measures the electric energy needed to extract an "infinitesimal" amount of charge from the mth body. $V_{m,n}$ is assumed to measure the energy needed to separate the bodies m and n to their respective positions. Thus, in particular we must have

$$V_{m,n} = V_m \qquad \text{on} \quad \partial\,\mathrm{supp}(q_m)$$

$$V_{n,m} = V_n \qquad \text{on} \quad \partial\,\mathrm{supp}(q_n)$$

Except for the context, these are the standard assumptions used to establish a solution of the electromagnetic field equations in the void.

5.1. The Classical Case

Let us assume that $f_n = 0$, and make a change from "Eulerian" to "Lagrangian" variables, i.e., interpret $y_n^i(t) = \varphi_n^i(x^i)$ as the position of a potential reference point in the nth body at the time $t > 0$ when (x^i) was its position at time $t = 0$. Moreover, if we then individualize the initial positions by putting

$$y_n^i(t) = \varphi_{nt}(x_n^i)$$

we obtain the Hamilton-Jacobi equation

$$\partial_t w + \sum_n \left[\frac{\delta^{ij}}{2m_n} (\partial_{x_n^i} w - A_{ni})(\partial_{x_n^j} w - A_{nj}) + e_n \frac{1}{2} \sum_{\substack{m \\ m \neq n}} V_{m,n} \right]$$

where

$$w(x_1^1, \ldots, x_N^i, t) = \tilde{w}(\varphi_{nt}^i(x_1^i), \ldots, \varphi_{nt}^i(x_N^i))$$

on $\mathcal{D}_V$. It is therefore quite reasonable to believe that if the extension of the bodies is small compared to the distance between them, the motions of the bodies follow the characteristics for the above Hamilton-Jacobi equation, i.e., satisfy the Hamilton equations

$$\dot{p}_{ni} = -\partial_{q_n^i} \mathcal{H}$$

$$\dot{q}_n^i = \partial_{p_{ni}} \mathcal{H}$$

for

$$\mathcal{H}(\cdot) = \sum_n \left[\frac{\delta^{ij}}{2m_n} (p_{ni} - e_n A_{ni})(p_{nj} - e_n A_{nj}) + e_n \frac{1}{2} \sum_{\substack{m \\ m \neq n}} V_{m,n} \right]$$

This would thus mean that the bodies essentially follow the standard classical motions.

5.2. The Quantum System

To describe a quantum system, we choose

$$f_n = -\frac{1}{2}\left(\frac{\hbar}{m_n}\right)^2 \left\{ \frac{\delta^{ij}}{\sqrt{\tilde{\varkappa}} \circ \Phi} [\varphi_{n,i}^k \varphi_{n,j}^l (\partial_{y_n^k} \partial_{y_n^l} \sqrt{\tilde{\varkappa}}) \circ \Phi + \varphi_{n,ij}^k (\partial_{y_n^k} \sqrt{\tilde{\varkappa}}) \circ \Phi \right.$$

$$\left. + \nabla_i \ln(j_{\varphi_n})\, \varphi_{n,j}^k (\partial_{y_n^k} \sqrt{\tilde{\varkappa}}) \circ \Phi] + \frac{1}{\sqrt{j_{\varphi_n}}} \nabla_i \nabla_j \sqrt{j_{\varphi_n}} \right\}$$

where $\tilde{\varkappa}$ is a "fixed" function

$$\tilde{\varkappa}: \quad \{(y_n^i) \in \mathbb{R}^{3N}\} \to \mathbb{R}_+$$

i.e.,

$$\chi^{\mathrm{H}}(\tilde{\varkappa}) = -\varphi^{j}_{n,i} v^{i}_{n} (\partial_{y^{j}_{n}} \tilde{\varkappa}) \circ \Phi$$

and

$$\Phi\colon \quad \mathbb{R}^{3N} \to \mathbb{R}^{3N}; \quad (\varphi_1, \ldots, \varphi_N) \mapsto (y_1, \ldots, y_N)$$

is defined by $y_1 = \varphi_1, \ldots, y_N = \varphi_N$.

Applying the same procedure as for the classical system, we obtain the equations

$$\partial_t w = -\left\{ \sum_n \left[\frac{\delta^{ij}}{2m_n} (\partial_{x^{i}_{n}} w - e_n A_{ni})(\partial_{x^{j}_{n}} - e_n A_{nj}) + e_n \frac{1}{2} \sum_{\substack{m \\ m \neq n}} V_{m,n} - \frac{1}{2} \frac{\hbar^2}{m_n} \frac{\delta^{ij}}{\sqrt{\varkappa}} \partial_{x^{i}_{n}} \partial_{x^{j}_{n}} \sqrt{\varkappa} \right] \right\}$$

$$\partial_t \varkappa = -\sum_n \partial_{x^{i}_{n}} \left[\varkappa \delta^{ij} \frac{1}{m_n} (\partial_{x^{j}_{n}} w - e_n A_{nj}) \right]$$

on $\mathcal{D}_{\mathrm{V}}$. Notice that

$$\varkappa(x_1, \ldots, x_N, t) = j_{\varphi_{1t}}(x_1) \ldots j_{\varphi_{Nt}}(x_N) \tilde{\varkappa}(\varphi_{1t}(x_1), \ldots, \varphi_{Nt}(x_N))$$

It is well known that these equations give rise to the Schrödinger equation

$$\partial_t \psi = \frac{1}{ih} \sum_n \left[\frac{\delta^{ij}}{2m_n} \left(\frac{\hbar}{i} \partial_{x^{i}_{n}} - e_n A_{ni} \right) \left(\frac{\hbar}{i} \partial_{x^{j}_{n}} - e_n A_{nj} \right) + e_n \frac{1}{2} \sum_{\substack{m \\ m \neq n}} V_{m,n} \right] \psi$$

for the wave function

$$\psi = \sqrt{\varkappa}\, e^{iw/\hbar}$$

In this case $w\colon \mathbb{R}^{3N} \to S^1$.

6. CONCLUSION

It is obvious that the theory thus presented is a direct generalization of quantum mechanics, and as such is based on the same experimental foundations. It is richer, however, and this permits us to make theoretical connections outside the scope of quantum mechanics. In particular, this theory opens up the possibility of giving a theoretical justification for the models used in atomic physics.

The theory might also have some consequences for the interpretation of quantum mechanics. In fact, since the very beginning the interpretation of quantum mechanics has been obscured by a lost dichotomy between matter and information, which was emphasized by the discussion between

Einstein and Bohr. The extension of the framework of quantum theory that is presented in this paper introduces this dichotomy and opens up the possibility of resolving some of the quantum mechanical paradoxes.

In this respect one may tentatively suggest that the matter aspect is entirely described by the fluid dynamical quantities, and that the f_n's or rather $\varkappa$ is determined by the "experimental conditions" under which the system is observed. If the expression "experimental conditions" is interpreted in a broad sense, including also, by transposition, the possibility that it is in some sense in the nature of a nontrivial quantum system to create these conditions by itself, this would explain why it is possible to claim that the wave function describes the state of the system.

It might be possible to solve some old and almost forgotten problems by applying the ideas presented above. However, this theory also gives rise to new problems. In particular, one would like to have a theoretical justification for the form chosen for f_n. In fact, our choice has been dictated by the result we wanted to obtain.

ACKNOWLEDGEMENT

I am indebted to C. Piron for numerous discussions on these subjects. I also thank the members of the Département de Physique Théorique for warm hospitality, and the Fonds National Suisse for support.

REFERENCES

Aaberge, T. (1986a). *Helvetica Physica Acta*, **59**, 390–409.
Aaberge, T. (1986b). Preprint, UGVA-DPT 1985/11-484.
Aaberge, T. (1987). *International Journal of Theoretical Physics*, **26**, 697–706.
Palais, R. S. (1968). *Foundations of Global Non-linear Analysis*, Benjamin, New York.
Pommaret, J. F. (1978). *Systems of Partial Differential Equations and Lie Pseudogroups*, Gordon and Breach, New York.